Annexure 'F'

SYLLABUS
CLASS –XII
(2013-14)

6. MATHEMATICS (Code No 041)

The Syllabus in the subject of Mathematics has undergone changes from time to time in accordance with growth of the subject and emerging needs of the society. Senior Secondary stage is a launching stage from where the students go either for higher academic education in Mathematics or for professional courses like engineering, physical and Bioscience, commerce or computer applications. The present revised syllabus has been designed in accordance with National Curriculum Frame work 2005 and as per guidelines given in Focus Group on Teaching of Mathematics 2005 which is to meet the emerging needs of all categories of students. Motivating the topics from real life situations and other subject areas, greater emphasis has been laid on application of various concepts.

Objectives

The broad objectives of teaching Mathematics at senior school stage intend to help the pupil:

- to acquire knowledge and critical understanding, particularly by way of motivation and visualization, of basic concepts, terms, principles, symbols and mastery of underlying processes and skills.
- to feel the flow of reasons while proving a result or solving a problem.
- to apply the knowledge and skills acquired to solve problems and wherever possible, by more than one method.
- to develop positive attitude to think, analyze and articulate logically.
- to develop interest in the subject by participating in related competitions.
- to acquaint students with different aspects of mathematics used in daily life.
- to develop an interest in students to study mathematics as a discipline.
- to develop awareness of the need for national integration, protection of environment, observance of small family norms, removal of social barriers, elimination of sex biases.
- to develop reverence and respect towards great Mathematicians for their contributions to the field of Mathematics.

CLASS-XII

One Paper	Three Hours	Marks: 100
Units		**Marks**
I.	RELATIONS AND FUNCTIONS	10
II.	ALGEBRA	13
III.	CALCULUS	44
IV.	VECTORS AND THREE - DIMENSIONAL GEOMETRY	17
V.	LINEAR PROGRAMMING	06
VI.	PROBABILITY	10
	Total	**100**

The Question Paper will include Value Based Question(s) to the extent of 3-5 marks

UNIT I. RELATIONS AND FUNCTIONS

1. Relations and Functions : (10) Periods

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

2. Inverse Trigonometric Functions: (12) Periods

Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

UNIT-II: ALGEBRA

1. Matrices: (18) Periods

Concept, notation, order, equality, types of matrices, zero matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants: (20) Periods

Determinant of a square matrix (up to 3 x 3 matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

UNIT-III: CALCULUS

1. **Continuity and Differentiability:** **(18) Periods**

Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit functions.Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.

2. **Applications of Derivatives:** **(10) Periods**

Applications of derivatives: rate of change of bodies, increasing/decreasing functions, tangents and normals, use of derivatives in approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. **Integrals:** **(20) Periods**

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, simple integrals of the following type to be evaluated.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$\int \frac{px+q}{ax^2 + bx + c} dx, \int \frac{px+q}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2-a^2} dx$$

$$\int \sqrt{ax^2 + bx + c} dx, \int (px+q)\sqrt{ax^2 + bx + c} dx.$$

Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. **Applications of the Integrals:** **(10) Periods**

Applications in finding the area under simple curves, especially lines, circles/parabolas/ ellipses (in standard form only), Area between the two above said curves (the region should be clearly identifiable).

5. **Differential Equations:** **(10) Periods**

Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx} + py = q$, where p and q are functions of x or constants

$\frac{dx}{dy} + px = q$, where p and q are functions of y or constants

UNIT-IV: VECTORS AND THREE-DIMENSIONAL GEOMETRY

1. **Vectors:** **(12) Periods**

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Scalar (dot) product of vectors, projection of a vector on a line. Vector (cross) product of vectors. Scalar triple product of vectors.

2. **Three - dimensional Geometry:** **(12) Periods**

Direction cosines and direction ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes. (iii) a line and a plane. Distance of a point from a plane.

UNIT-V: LINEAR PROGRAMMING

1. **Linear Programming:** **(12) Periods**

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

UNIT-VI: PROBABILITY

1. **Probability:** **(18) Periods**

   Conditional probability, multiplication theorem on probability. independent events, total probability, Baye's theorem, Random variable and its probability distribution, mean and variance of random variable. Repeated independent (Bernoulli) trials and Binomial distribution.

Recommended Textbooks.

1) Mathematics Part I - Textbook for Class XI, NCERT Publication
2) Mathematics Part II - Textbook for Class XII, NCERT Publication
3) Laboratory Manual Mathematics (Higher Secondary Stage) NCERT Publication

SAMPLE QUESTION PAPER

MATHEMATICS

CLASS –XII (2013-14)

BLUE PRINT

Unit		**VSA (1)**	**SA (4)**	**LA (6)**	**Total**	
I.	Relations and Functions	1 (1)	4 (1)	–	5 (2)	10 (4)
	Inverse Trigonometric Functions	1 (1)	*4 (1)	–	5 (2)	
II.	Matrices	2 (2)	–	VB 6 (1)	8 (3)	13 (5)
	Determinants	1 (1)	4 (1)	–	5 (2)	
III.	Continuity and Differentiability	–	8 (2)	–	18 (4)	44 (11)
	Application of Derivatives	–	*4 (1)	6 (1)		
	Integrals	2 (2)	*4 (1)	6 (1)	18 (5)	
	Application of Integrals	–	–	6 (1)	–	
	Differential equations	–	8 (2)	–	8 (2)	
IV.	Vectors	2 (2)	*4 (1)	–	6 (3)	17 (6)
	3–dim geometry	1 (1)	4 (1)	*6 (1)	11 (3)	
V.	Linear Programming	–	–	VB 6 (1)	–	6 (1)
VI.	Probability	–	VB 4 (1)	*6 (1)	–	10 (2)

SECTION–A

Question number 1 to 10 carry 1 mark each.

1. Write the smallest equivalence relation R on Set A = {1, 2, 3}.

2. If $|\vec{a}| = a$, then find the value of $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$

3. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x–axis at angles 30^0 and 120^0 respectively, then write the value of $|\vec{a} + \vec{b}|$

4. Find the sine of the angle between the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane 2x – 2y + z – 5 = 0.

5. Evaluate $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

6. If A = $\begin{pmatrix} 4 & 6 \\ 7 & 5 \end{pmatrix}$, then what is A. (adj. A)?

7. For what value of k, the matrix $\begin{pmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{pmatrix}$ is skew symmetric?

8. If $\begin{vmatrix} \sin\alpha & \cos\beta \\ \cos\alpha & \sin\beta \end{vmatrix} = \frac{1}{2}$, where α, β are acute angles, then write the value of α+β.

9. If $\int_0^1 (3x^2 + 2x + k)dx = 0$, write the value of k.

10. Evaluate: $\int \cot x\,(\operatorname{cosec} x - 1)e^x dx$.

SECTION–B

Questions numbers 11 to 22 carry 4 marks each.

11. Let S be the set of all rational numbers except 1 and * be defined on S by a * b = a + b – ab, ∀ a, b εS. Prove that:

    a) * is a binary on S.

    b) * is commutative as well as associative. Also find the identity element of *.

12. If a + b + c ≠ o and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = o$, then using properties of determinants, prove that a = b = c.

13. Evaluate: $\int (2\sin 2x - \cos x)\sqrt{6 - \cos^2 x - 4\sin x}\ dx$

OR

Evaluate: $\int \frac{5x}{(x+1)(x^2+9)} dx$

14. Find a unit vector perpendicular to the plane of triangle ABC where the vertices are A (3, –1, 2) B (1,–1, –3) and C (4, –3, 1).

OR

Find the value of λ, if the points with position vectors $3\hat{i} - 2\hat{j} - \hat{k}$, $2\hat{i} + 3\hat{j} - 4\hat{k}$, $-\hat{i} + \hat{j} + 2\hat{k}$ and $4\hat{i} + 5\hat{j} + \lambda\hat{k}$ are coplanar.

15. Show that the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda\,(3\hat{i} - \hat{j})$ and $\vec{r} = (4\,\hat{i} - \hat{k}) + \mu\,(2\hat{i} + 3\,\hat{k})$ intersect. Also find their point of intersection.

16. Prove that $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$.

OR

Find the greatest and least values of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$.

17. Show that the differential equation $x\text{dy} - \text{y}dx = \sqrt{x^2 + y^2}\, dx$ is homogeneous, and solve it.

18. Find the particular solution of the differential equation cos x dy = sin x (cos x – 2y) dx, given that y = 0 when $x = \frac{\pi}{3}$.

19. Out of a group of 8 highly qualified doctors in a hospital, 6 are very kind and cooperative with their patients and so are very popular, while the other two

remain reserved. For a health camp, three doctors are selected at random. Find the probability distribution of the number of very popular doctors.

What values are expected from the doctors?

20. Show that the function g(x) = $|x-2|$, $x \in$R, is continuous but not differentiable at $x = 2$.

21. Differentiate $\log\left(x^{\sin x} + \cot^2 x\right)$ with respect to x.

22. Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

OR

Separate the interval $\left[0, \frac{\pi}{2}\right]$ into sub–intervals in which f (x) = $\sin^4 x + \cos^4 x$ is increasing or decreasing.

SECTION–D

Question numbers 23 to 29 carry 6 marks each.

23. Find the equation of the plane through the points A (1, 1, 0), B (1, 2, 1) and C (–2, 2, –1) and hence find the distance between the plane and the line $\frac{x-6}{3} = \frac{y-3}{-1} = \frac{z+2}{1}$.

OR

A plane meets the x, y and z axes at A, B and C respectively, such that the centroid of the triangle ABC is (1, –2, 3). Find the Vector and Cartesian equation of the plane.

24. A company manufactures two types of sweaters, type A and type B. It costs Rs. 360 to make one unit of type A and Rs. 120 to make a unit of type B. The company can make atmost 300 sweaters and can spend atmost Rs. 72000 a day. The number of sweaters of type A cannot exceed the number of type B by more than 100. The company makes a profit of Rs. 200 on each unit of type A but considering the

difficulties of a common man the company charges a nominal profit of Rs. 20 on a unit of type B. Using LPP, solve the problem for maximum profit.

25. Evaluate: $\int_0^1 x\left(\tan^{1-}x\right)^2 \mathrm{d}x$

26. Using integration find the area of the region $\left\{(x, \mathrm{y}): x^2+y^2 \leq 1 \leq x+\frac{y}{2}, x, y \in R\right\}$.

27. A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of three types of seeds are 45%, 60% and 35%. Calculate the probability.

a) of a randomly chosen seed to germinate.

b) that it is of the type A_2, given that a randomly chosen seed does not germinate.

OR

Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then two balls are drawn at random (without replacement) from Bag II. The balls so drawn are found to be both red in colour. Find the probability that the transferred ball is red.

28. Two schools A and B want to award their selected teachers on the values of honesty, hard work and regularity. The school A wants to award Rs. x each, Rs. y each and Rs. z each for the three respective values to 3, 2 and 1 teachers with a total award money of Rs. 1.28 lakhs. School B wants to spend Rs. 1.54 lakhs to award its 4, 1 and 3 teachers on the respective values (by giving the same award money for the three values as before). If the total amount of award for one prize on each value is Rs. 57000, using matrices, find the award money for each value.

29. A given rectangular area is to be fenced off in a field whose length lies along a straight river. If no fencing is needed along the river, show that the least length of fencing will be required when length of the field is twice its breadth.

MARKING SCHEME

SECTION–A

1. R = {(1,1), (2, 2), (3, 3)} 1

2. $2a^2$ 1

3. $\sqrt{2}$ 1

4. $\frac{1}{5\sqrt{2}}$ 1

5. $-\frac{\pi}{3}$ 1

6. $\begin{pmatrix} -22 & 0 \\ 0 & -22 \end{pmatrix}$ 1

7. $k = -\frac{3}{2}$ 1

8. $\frac{2\pi}{3}$ 1

9. k = –2 1

10. –cot x e^x + C 1

SECTION–B

11. a) let $a_1, a_2 \in S$

$\therefore a_1 * a_2 = a_1 + a_2 - a_1 a_2$

Since $a_1 \neq 1, a_2 \neq 1 \Rightarrow (a_1 - 1)(a_2 - 1) \neq 0$

$\Rightarrow a_1 a_2 - a_1 - a_2 + 1 \neq 0$ or $a_1 + a_2 - a_1 a_2 \neq 1$

$\Rightarrow a_1 * a_2 \in$ S $\therefore *$ is a binary. 1

b) $a_2 * a_1 = a_2 + a_1 + a_1 a_2 = a_1 * a_2 \Rightarrow *$ is commutative 1

also, $(a_1 * a_2) * a_3 = (a_1 + a_2 - a_1 a_2) * a_3$

$= a_1 + a_2 - a_1 a_2 + a_3 - a_1 a_3 - a_2 a_3 + a_1 a_2 a_3$

$a_1 * (a_2 * a_3) = a_1 * \left(a_2 + a_3 - a_2a_3\right)$

$= a_1 + a_2 + a_3 - a_2a_3 - a_1a_2 - a_1a_3 + a_1a_2a_3$

$(a_1 * a_2) * a_3 = a_1 * (a_2 * a_3)$ i.e. $*$ is associative 1

Let e be the identity,

Then a * e = $a \Rightarrow a$ + e – ae = a $\Rightarrow$ e (1–a) = 0 1

Since 1 – $a \neq 0 \Rightarrow$ e = 0.

12. $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$C_1 \rightarrow C_1 + C_2 + C_3 \quad \therefore \Delta = (a + b + c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$ 1

$R_2 \rightarrow R_2 - R_1,\ R_3 \rightarrow R_3 - R_1 \quad \therefore \Delta = (a + b + c) \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$

$= (a + b + c)\ (-b^2 - c^2 + 2bc - a^2 + ac + ab - bc) = 0$ 1

$\Delta = -\ (a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$= -\frac{1}{2}(a + b + c)\ [(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$ 1

Since $a + b + c = 0 \Rightarrow a-b = 0,\ b-c = 0,\ c-a = 0$

or a = b = c. 1

13. $I = \int (2\sin 2x - \cos x)\left(\sqrt{6 - \cos^2 x - 4\sin x}\right) dx$

$= \int (4\sin x - 1)\left(\sqrt{\sin^2 x - 4\sin x + 5}\right) \cos x\ dx$ 1

$= \int (4t - 1)\sqrt{t^2 - 4t + 5}\ dt$ where $\sin x = t$ ½

$= 2\int(2t-4)\sqrt{t^2-4t+5}\,dt + 7\int\sqrt{(t-2)^2+1}\,dt$ ½

$= 2\frac{(t^2-4t+5)^{3/2}}{\frac{3}{2}} + 7\left[\frac{t-2}{2}\sqrt{t^2-4t+5} + \log\left|(t-2)+\sqrt{t^2-4t+5}\right| + c\right]$ 1

$= \frac{4}{3}\left[\sin^2 x - 4\sin x + 5\right]^{3/2} + 7\frac{(\sin x - 2)}{2}$

$\left[\sqrt{\sin^2 x - 4\sin x + 5} + \log\left|(\sin x - 2)\right| + \sqrt{\sin^2 x - 4\sin x + 5}\right] + C$ 1

OR

$I = \int\frac{5x}{(x+1)(x^2+9)}dx$

$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+c}{x^2+9} \Rightarrow A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{9}{2}$ 2

$\Rightarrow I = -\frac{1}{2}\int\frac{1}{x+1}dx + \frac{1}{2}\int\frac{x+9}{x^2+9}dx$ 1

$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\frac{x}{3} + C$ 1

14. A vector perpendicular to the plane of ΔABC

$= \overrightarrow{AB} \times \overrightarrow{BC}$ ½

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 3 & -2 & 4 \end{vmatrix} = -10\hat{i} - 7\hat{j} + 4\hat{k}$ 1+1

or $10\hat{i} + 7\hat{j} - 4\hat{k}$

$\left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = \sqrt{100+49+16} = \sqrt{165}$ 1

∴ Unit vector ⊥ to plane of ABC $= \frac{1}{\sqrt{165}}(10\hat{i} + 7\hat{j} - 4\hat{k})$ ½

OR

Let the points be A (3, –2, –1), B (2, 3, –4), C (–1, 1, 2) and D (4, 5, λ)

$\therefore \overrightarrow{AB} = -\hat{\imath} + 5\hat{\jmath} - 3\hat{k}$,

$\overrightarrow{AC} = 4\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$

$\overrightarrow{AD} = \hat{\imath} + 7\hat{\jmath} + (\lambda + 1)\hat{k}$ 1½

A, B, C, D are coplanar if $\left[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\right] = 0$ ½

$$\left[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\right] = \begin{vmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \\ 1 & 7 & \lambda + 1 \end{vmatrix} = 0$$ ½

$\therefore$ 1(15+9) – 7 (–3 – 12) + (λ+1) (–3+20) = 0 1

$\Rightarrow \lambda = -\frac{146}{17}$ ½

15. Let, any point on the line $\vec{r} = \left(\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \lambda(3\hat{\imath} - \hat{\jmath})$ be P $(1 + 3\lambda, 1 - \lambda, -1)$ ½

and any point on line $\vec{r} = \left(4\hat{\imath} - \hat{k}\right) + \mu(2\hat{\imath} + 3\hat{k})$ be Q $(4 + 2\mu, 0, -1 + 3\mu)$ ½

If the lines intersect, P and Q must coincide for some λ and μ. ½

$\therefore$ 1+3λ = 4+2μ ..... (i)

1–λ = 0 ..... (ii)

–1 = –1 + 3μ ..... (iii)

Solving (ii) and (iii) we get λ = 1 and μ = 0 1

Putting in (i) we get 4 = 4, hence lines intersect. ½

$\therefore$ P or Q (4, 0, –1) is the point of intersection. 1

16. LHS = $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18}$ ½

$= \tan^{-1}\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}.\frac{1}{8}} + \tan^{-1}\frac{1}{18}$ 1

$= \tan^{-1}\frac{3}{11} + \tan^{-1}\frac{1}{18}$ ½

$= \tan^{-1}\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}.\frac{1}{18}} = \tan^{-1}\frac{65}{195} = \tan^{-1}\frac{1}{3}$ 1½

$= \cot^{-1} 3$=RHS ½

OR

$(\sin^{-1}x)^2 + (\cos^{-1}x)^2 = \left(\sin^{-1}x + \cos^{-1}x\right)^2 - 2\sin^{-1}x\cos^{-1}x$ ½

$= \left(\frac{\pi}{2}\right)^2 - 2\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)$ ½

$= \frac{\pi^2}{4} - \pi\sin^{-1}x + 2\left(\sin^{-1}x\right)^2$

$= 2\left[\left(\sin^{-1}x\right)^2 - \frac{\pi}{2}\sin^{-1}x + \frac{\pi^2}{8}\right]$

$= 2\left[\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$ 1

$\therefore$ least value = $2\left[\frac{\pi^2}{16}\right] = \frac{\pi^2}{8}$ 1

and greatest value = $2\left[\left(\frac{-\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right] = \frac{5\pi^2}{4}$ 1

17. $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x} = \frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2} = f\left(\frac{y}{x}\right)$ ½

$x \to \lambda x$, $y \longrightarrow \lambda y \Rightarrow \frac{dy}{dx} = \frac{\lambda y}{\lambda x} + \sqrt{1+\left(\frac{\lambda x}{\lambda y}\right)^2} = \lambda^0\left[\frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2}\right]$ 1

$= \lambda^0 . f(y/x)$

∴ differential equation is homogeneous.

let $\frac{y}{x}$ = v or y = vx ∴ $\frac{dy}{dx}$ = v + $x\frac{dv}{dx}$ ½

v+$x\frac{dv}{dx}$ = v+$\sqrt{1+v^2}$ or $\int\frac{dx}{\sqrt{1+v^2}} = \int\frac{dx}{x}$ 1

⟹ log|v + $\sqrt{1+v^2}$ | = log cx ⟹ v + $\sqrt{1+v^2}$ = cx 1

⟹ y + $\sqrt{x^2+y^2}$ = cx^2 1

18. Given differential equation can be written as

cot $x\frac{dy}{dx}$ + 2y = cos x or $\frac{dy}{dx}$ + 2 tan xy = sin x ½

⟹ Integrating factor = $e^{\int 2tanxdx} = e^{2logsec\,x}$ = sec² x. 1

∴ the solution is y. sec²x = $\int$ sin $x.sec^2x$ dx ½

= $\int secx.tanx\ dx$

y. sec²x = sec x + c 1

⟹ y = cos$x$ + c cos²x.

When $x = \frac{\pi}{3}$, y = 0 ⟹ 0 = $\frac{1}{2} + \frac{1}{4}$C ⟹ C = -2 ½

Hence the solution is y = cos$x$–2cos² x. ½

19. let x be the random variable representing the number of very popular doctors.

∴ **x :**	**1**	**2**	**3**	
P(x)	$\frac{{}^6C_1 . {}^2C_2}{{}^8C_3}$	$\frac{{}^6C_2 . {}^2C_1}{{}^8C_3}$	$\frac{{}^6C_3}{{}^8C_3}$	**½**
	$= \frac{3}{28}$	$= \frac{15}{28}$	$= \frac{10}{28}$	**1½**

It is expected that a doctor must be

- **Qualified**
- **Very kind and cooperative with the patients** } **2**

20. $g(x) = |x-2| = \begin{cases} x-2, & x \geq 2 \\ 2-x, & x < 2 \end{cases}$ ½

LHL = $\lim_{x \to 2^-} (2-x) = 0$

RHL = $\lim_{x \to 2^+} (x-2) = 0$ and g (2) = 0 } 1

∴ g(x) is continuous at x = 2…………………………….. ½

LHD = $\lim_{h \to 0} \frac{g(2)-g(2-h)}{h} = \lim_{h \to 0} \frac{0-(2-2+h)}{h} = -1$ 1

RHD = $\lim_{h \to 0} \frac{g(2+h)-g(2)}{h} = \lim_{h \to 0} \frac{(2+h-2)-0}{h} = 1$ ½

LHD ≠ RHD ∴ g(x) is not differentiable at x = 2 ½

21. Let y = log ($x^{\sin x} + \cot^2 x$)

$\Rightarrow \frac{dy}{dx} = \frac{1}{x^{\sin x} + Cot^2 x} \frac{d}{dx} (x^{\sin x} + \cot^2 x)$…………………….. 1

Let u = x^{sinx} and v = $\cot^2 x$.

∴ log u = sin$x$. log x , $\frac{dv}{dx} = 2 \cot x\,(-\text{cosec}^2 x)$ ½

$\frac{1}{u}\frac{du}{dx} = \frac{sinx}{x} + \log x.\cos x$

or $\frac{du}{dx} = x^{\sin x}\left[\frac{sinx}{x} + \cos x \log x\right]$ ……………………………….. 1

∴ $\frac{dy}{dx} = \frac{1}{x^{sinx} + cot^2 x}\left[x^{\sin x}\left(\frac{sinx}{x} + \text{cos}x\; logx\right) - 2\; cotxcosec^2\, x\right]$ 1 ½

22. Solving $xy = a^2$ and $x^2+y^2 = 2a^2$ to get $x = \pm a$

$\therefore$ for x = a, y = a and x = -a, y = -a

i.e the two curves intersect at P (a, a) and Q (-a, -a) 1

$xy = a^2 \Rightarrow x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x} = -1$ at P and Q 1

$x^2+y^2 = 2a^2 \Rightarrow 2x+2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} = -1$ at P and Q 1

$\therefore$ Two curves touch each other at P

as well as at Q. 1

OR

$f(x) = \sin^4 x + \cos^4 x$

$f'(x) = 4\sin^3 x\cos x - 4\cos^3 x\sin x$

$= -4\sin x\cos x(\cos^2 x - \sin^2 x)$

$= -2\sin 2x\cos 2x = -\sin 4x$ 1

$f'(x) = 0 \Rightarrow \sin 4x = 0 \Rightarrow 4x = 0, \pi, 2\pi, 3\pi, \ldots\ldots$

$x = 0, \frac{\pi}{4}, \frac{\pi}{2}$ 1

Sub Intervals are $\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\therefore f'(x) < 0$ in $(0, \pi/4)$ $\therefore$ f(x) is decreasing in $(0, \pi/4)$ 1

And $f'(x) > 0$ in $(\pi/4, \pi/2)$ $\therefore$ f(x) in increasing in $(\pi/4, \pi/2)$ 1

SECTION – D

23. A vector ⊥ to the plane is parallel to $\overrightarrow{AB} \times \overrightarrow{BC}$ 1

$\therefore \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ -3 & 0 & -2 \end{vmatrix} = -2\hat{i} - 3\hat{j} + 3\hat{k}$ or $2\hat{i} + 3\hat{j} - 3\hat{k}$ 1½

∴ Equation of plane is $\vec{r}.(2\hat{i} + 3\hat{j} - 3\hat{k}) = 5$ 1

$(\vec{r}.\vec{n} = \vec{a}.\vec{n})$

Since, $(2\hat{i} + 3\hat{j} - 3\hat{k}).(3\hat{i} - \hat{j} + \hat{k}) = 0$, so the given line is parallel to the plane. ½

∴ Distance between the point (on the line) (6, 3, -2) and the plane $\vec{r}.$

$(2\hat{i} + 3\hat{j} - 3\hat{k}) - 5 = 0$ is

$$d = \frac{|12+9+6-5|}{\sqrt{4+9+9}} = \frac{22}{\sqrt{22}} = \sqrt{22}$$ 1½

Let the coordinates of points A, B and C be (a, 0, 0), (0, b, 0) and (0, 0, c) respectively. ½

∴ Equation of plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and 1

$\frac{a+0+0}{3} = 1$, $\frac{0+b+0}{3} = -2$ and $\frac{0+0+c}{3} = 3$ 1½

$\Rightarrow$ a = 3, b = -6 and c = 9

∴ Equation of plane is $\frac{x}{3} + \frac{y}{-6} + \frac{z}{9} = 1$ 1

or $6x - 3y + 2z - 18 = 0$ 1

which in vector form is

$$\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) = 18$$ 1

24. let the company manufactures sweaters of type A = x, and that of type B = y. daily

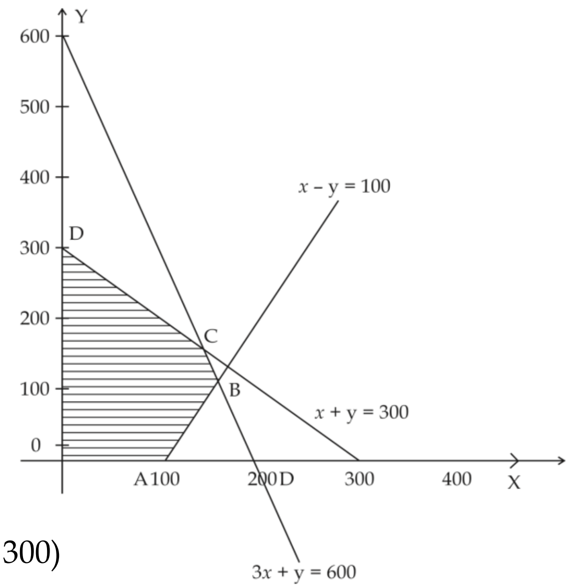
∴ LPP is Maximise P = $200x + 20y$ 1

s.t. $x+y \leq 300$

$360x + 120y \leq 72000$

$x - y \leq 100$ 1½

$x \geq 0$ $y \geq 0$



Correct Graph 1½

Getting vertices of the feasible region as

A (100, 0), B (175, 75), C (150, 150) and D (0, 300)

Maximum profit at B

So Maximum Profit = 200 (175) + 20 (75)

= 35000 + 1500 1½

= Rs. 36500

25. let I $= \int_0^1 (\tan^{-1}x)^2 . x \, dx$

$= \left[(\tan^{-1}x)^2 . \frac{x^2}{2}\right]_0^1 - \int_0^1 2\tan^{-1}x . \frac{1}{1+x^2} \frac{x^2}{2} \, dx$ 1

$= \frac{\pi^2}{32} - \int_0^1 \tan^{-1}x . \frac{x^2}{1+x^2} \, dx$ ½

$x = \tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$ ½

$= \frac{\pi^2}{32} - \int_0^{\pi/4} \theta . \tan^2\theta \, d\theta$

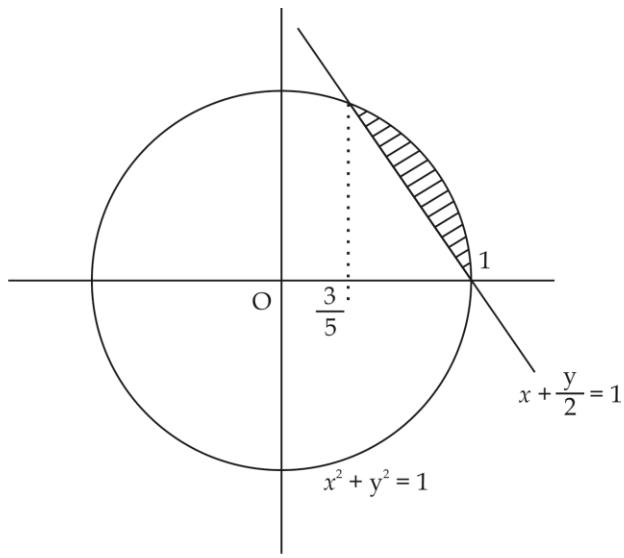
$= \frac{\pi^2}{32} - \int_0^{\pi/4} \theta . \sec^2\theta \, d\theta + \int_0^{\pi/4} \theta . d\theta$ 1

$= \quad \frac{\pi^2}{32} - [\theta\ tan\theta]_0^{\pi/4} \int_0^{\pi/4} tan\ \theta\ d\theta + \left[\frac{\theta^2}{2}\right]_0^{\pi/4}$ 1

$= \quad \frac{\pi^2}{32} - \frac{\pi}{4} + [\log \sec \theta]_0^{\pi/4} + \frac{\pi^2}{32}$ 1

$= \quad \frac{2\pi^2}{32} - \frac{\pi}{4} + \frac{1}{2}\ log2\ or\ \frac{\pi^2-4\pi}{16} - \frac{1}{2}\log 2$ 1

26. Correct figure: 1



Correct Figure 1

Solving $x^2 + y^2 = 1$ and $x + \frac{y}{2} = 1$ to get $x = \frac{3}{5}$ and $x = 1$ as points of intersection

Required area = $\int_{3/5}^{1} \sqrt{1-x^2}\ dx - \int_{3/5}^{1} (2-2x)dx$ 1

$= \quad \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\ sin^{-1}x\right]_{3/5}^{1} - [2x - x^2]_{3/5}^{1}$ 1

$= \quad \frac{\pi}{4} - \left(\frac{6}{25} + \frac{1}{2}\ sin^{-1}\ \frac{3}{5}\right) - \left[1 - \frac{21}{25}\right]$ 1

$= \quad \left(\frac{\pi}{4} - \frac{2}{5} - \frac{1}{2}\ sin^{-1}\ \frac{3}{5}\right)$ sq. u. 1

27. let E_1 : randomly selected seed is A_1 type $P(E_1) = \frac{4}{10}$

E_2 : randomly selected seed is A_2 type $P(E_2) = \frac{4}{10}$

E_3 : randomly selected seed is A_3 type $P(E_3) = \frac{2}{10}$ 1

(i) let A : selected seed germinates

$\therefore P(A/E1) = \frac{45}{100}, P(A/E_2) = \frac{60}{100}, P(A/E3) = \frac{35}{100}$ 1

$\therefore P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3).\,P(A/E_3)$ ½

$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$

$= \frac{49}{100}$ or 0.49 1

(ii) let A : selected seed does not germinate ½

$\therefore P(A/E_1) = \frac{55}{100}, P(A/E_2) = \frac{40}{100}, P(A/E_3) = \frac{65}{100}$ ½

$\therefore P(E_2/A) = \frac{P(E_2).P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)\,P(A|E_2)+\,P(E_2)\,P(A|E_3)}$ ½

$= \frac{\frac{4}{10} \times \frac{40}{100}}{\frac{4}{10} \times \frac{55}{10} + \frac{4}{10} \times \frac{40}{100} + \frac{2}{10} \times \frac{65}{100}} = \frac{16}{51}$ 1

OR

Let E_1 : transferred ball is red.

E_2 : transferred ball is black. ½

A : Getting both red from 2nd bag (after transfer)

$P(E_1) = \frac{3}{7}$ $P(E_2) = \frac{4}{7}$ 1

$P(A/E1) = \frac{{}^5C_2}{{}^{10}C_2} = \frac{10}{45}$ or $\frac{2}{9}$ 1

$P(A/E2) = \frac{{}^4C_2}{{}^{10}C_2} = \frac{6}{45}$ or $\frac{2}{15}$ 1

$P(E_1/A) = \frac{P(E_1).P(A|E_1)}{P(E1).P(A|E_1)+P(E_2)\,P(A|E_2)}$ ½

$= \frac{\frac{3}{7}\cdot\frac{2}{9}}{\frac{3}{7}\cdot\frac{2}{9}+\frac{4}{7}\cdot\frac{2}{15}} = \frac{5}{9}$ 1+1

28. The three equations are $3x+2y+z = 1.28$

$$\left.\begin{aligned} 4x + y + 3z &= 1.54 \\ x + y + z &= 0.57 \end{aligned}\right\}$$ 1½

$\Rightarrow \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.28 \\ 1.54 \\ 0.57 \end{pmatrix}$ i.e AX = B ½

$|A| = -5$ and $X = A^{-1}B$ 1

$A^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{pmatrix}$ 1

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{pmatrix}\begin{pmatrix} 1.28 \\ 1.54 \\ 0.57 \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.21 \\ 0.11 \end{pmatrix}$ ½

$x = 25000$, $y = 21000$, $z = 11000$ 1½

29. let length be x m and breadth be y m.

∴ length of fence $L = x+2y$

Let given area = a $\Rightarrow xy = a$ or $y = \frac{a}{x}$

$\Rightarrow L = x + \frac{2a}{x}$ 1

$\frac{dL}{dx} = 1 - \frac{2a}{x^2}$ 1

$\frac{dL}{dx} = 0 \implies x^2 = 2a \quad \therefore x = \sqrt{2a}$ 1

$\frac{d^2L}{dx^2} = \frac{2a}{x^3} > 0$ ½

$\implies$ for minimum length L = $\sqrt{2a} + \frac{2a}{\sqrt{2a}} = 2\sqrt{2a}$ 1

$x = \sqrt{2a}$ and breadth y = $\frac{a}{\sqrt{2a}} = \frac{\sqrt{2a}}{2} = \frac{1}{2}x$ 1

$\implies x = 2y$ ½